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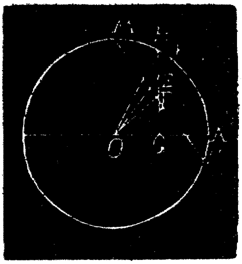
Let P be the random point. Through P draw HG perpendicular to 'A. Let $OA=a$, $GP=x$, $\angle HOA=\theta$, area circle required$=\frac{\pi}{4}OP^2=\frac{\pi}{4}(x^2+a^2\cos^2\theta)$. An element of the circle at P, is $a\sin\theta d\theta dx$. The limits of θ are 0 and 2π; of x, 0 and $a\sin\theta=x'$. $\triangle$=average area.



$$\therefore\ \triangle=\frac{\int_0^{2\pi}\int_0^{x'}\frac{\pi}{4}(x^2+a^2\cos^2\theta)a\sin\theta d\theta dx}{\int_0^{2\pi}\int_0^{x'}a\sin\theta d\theta dx}$$

$$=\frac{1}{4a}\int_0^{2\pi}\int_0^{x'}(x^2+a^2\cos^2\theta)\sin\theta d\theta dx=\frac{a^2}{12}\int_0^{2\pi}(\sin^2\theta+3\cos^2\theta)\sin^2\theta d\theta=\frac{\pi a^2}{8}.$$

II. Solution by the PROPOSER.

Let $OP=x$; then the average area of the circle whose diameter is OP,

becomes $A=\int_0^{2\pi}\int_0^{r}(\frac{1}{4}\pi x^2)xd\theta dx\div\int_0^{2\pi}\int_0^{r}xd\theta dx=\frac{1}{8}\pi r^2$.

Professor Zerr furnished five different solutions of this problem; Professor Matz seven and Mr. Dolman three.

◆

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

15. Proposed by SAMUER HART WRIGHT, M.D., M.A.,Ph.D., Penn Yan, Yates Co., New York.

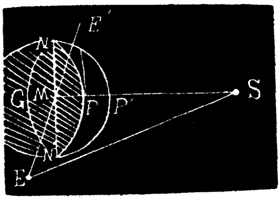
Required the illuminated area of the Moon's disc when ⅔ through its first quarter of 60° of longitude east of the Sun, the Earth and Moon being at their mean distances.

II. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton. Virginia, and the PROPOSER.

A rigorous solution of this problem would present many difficulties. In the first place, the surface of the Moon is more than half illuminated; in the second, no observer sees half of the Moon's surface at one time, in the third, the mean distance of the Sun is known within a quarter million miles.

In the following solution we will assume half the Moon's surface illuminated, and half of its surface as presented to an observer at one time. Also we will take the Sun's parallax 8.″81 and hence, his mean distance as 149320000 kilometers, and the Moon's mean distance as 384000 kilometers.

Let S, M, E, be the centres of the Sun, Moon, and Earth respectively. Let $EM=1$, then $SE=388.854$, also $\angle MES=60^\circ$.



From Trigonometry we get

$$EM+ES:ES-EM=\tan\tfrac{1}{2}(EMS+ESM):\tan\tfrac{1}{2}(EMS-ESM)$$

$$389.854:387.854=\tan 60^\circ:\tan\tfrac{1}{2}(EMS-ESM).$$

$$\therefore\ \tfrac{1}{2}(EMS-ESM)=59^\circ 52'20'',$$

$$\therefore\ \angle EMS=119^\circ 52'20''.$$

Now since EM is perpendicular to the plane $N'BNP'$, and MS is perpendicular to the plane $N'GNP$, $\angle P'ME=180^\circ-119^\circ 52'20''=60^\circ 7'40''=\theta$.

Let r=radius of the Moon, then MP the semi-conjugate diameter of the ellipse $N'GNP=r\cos\theta$.

Area of bright crescent $N'PNP'N'$=area of semi-ellipse $NMNPN'$ subtracted from semi-circle $N'NPN'$; area semi-circle$=\frac{1}{2}\pi r^2$,

area semi ellipse$=\frac{1}{2}\pi r^2\cos\theta$; and area crescent$=\frac{1}{2}\pi r^2(1-\cos\theta)=.250965\pi r^2$ $=\frac{1}{4}$ of the disc nearly. In the above solution we have regarded the observer as being at the centre of the earth.

16. Yale Senior Prize Problem. Contributed by H. A. NEWTON, LL.D., Professor of Mathematics, Yale College, New Haven, Connecticut.

The axis of two right cylinders whose bases are circles of 4 and 6 inches radius respectively, intersect at right angles; compute to four decimal places the lengths of the curves of intersection of the two surfaces.

Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $x^2+y^2=36$, be the equation to the cylinder with base radius 6 inches.

$x^2+z^2=16$, the equation to the cylinder with base radius 4 inches.

$$\text{Then}\ \frac{dy}{dx}=-\frac{x}{\sqrt{(36-x^2)}},\ \frac{dz}{dx}=-\frac{x}{\sqrt{(16-x^2)}}.$$

$$\therefore\ S=\int\left\{1+\left(\frac{dy}{dx}\right)^2+\left(\frac{dz}{dx}\right)^2\right\}^{\frac{1}{2}}dx=\int\left\{\frac{576-x^4}{(36-x^2)(16-x^2)}\right\}^{\frac{1}{2}}dx.$$

The whole length of the curve$=4s=4\int_0^4\left\{\frac{576-x^4}{(36-x^2)(16-x^2)}\right\}^{\frac{1}{2}}dx.$

Let $x=4\sin\theta$.

$$\text{Then}\ 4s=16\int_0^{\frac{\pi}{2}}\left\{\frac{9-4\sin^4\theta}{9-4\sin^2\theta}\right\}^{\frac{1}{2}}d\theta=16\int_0^{\frac{\pi}{2}}\left\{\frac{1-e^2\sin^4\theta}{1-e^2\sin^2\theta}\right\}^{\frac{1}{2}}d\theta,\ \text{where}\ e=\tfrac{2}{3}.$$

Expanding we get

$$4s=16\int_0^{\frac{\pi}{2}}(1+\tfrac{2}{9}\sin^2\theta-\tfrac{4}{27}\sin^4\theta-\tfrac{16}{729}\sin^6\theta-\tfrac{200}{6561}\sin^8\theta-\tfrac{48}{6561}\sin^{10}\theta$$
$$-\tfrac{1408}{177147}\sin^{12}\theta-\tfrac{3392}{1594323}\sin^{14}\theta-\ldots.)d\theta.$$